Motion in a Viscous Medium

1. Introduction

Oatmeal yogurt cup, shown in picture 1, is a popular low-fat food around teenagers recently. Comparing with traditional oats porridge, the colourful fruits and clear slayers make oatmeal yogurt cup look more delightful. I noticed that pouring oat into yogurt usually takes longer time to sink than in milk, and this characteristic is the core to make a beautiful yogurt cup. Therefore, I decided to design an experiment to figure out the motion in a viscous medium.

Picture 1: Oatmeal yogurt cup

To do so, I choose detergent as viscous medium instead of testing yogurt directly, since firstly yogurt is not transparent, which increases difficulty in experiment; secondly after experiment, the detergent could still be used but yogurt will be wasted. The experiment is constituted by two parts: part 1 mainly focuses on verifying the existence of buoyancy force and fluid resistance, and to determine the direction of these forces; part 2 tests whether it follows the Stoke's Law.

2. Theory

The force exerted by a fluid (i.e. gas or a liquid) on a body moving through it is known as fluid resistance. The moving body exerts a force on the fluid to push it out of the way. The fluid, according to Newton's Third Law, pushes back the body with an equal and opposite force.

Consider the case of a sphere falling in a viscous fluid (see Figure 1). The gravity force acting on the sphere is **mg**, the buoyancy force is **B**, and the fluid resistance is **f**. The force **f** is always in the opposite direction of the sphere's velocity relative to the fluid. Usually, force **f** increases as the velocity of the sphere increases,

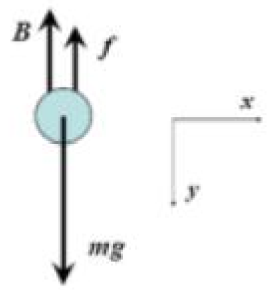


Figure 1: Free-body diagram for a sphere falling downward through a fluid.

In 1851, George Gabriel Stokes derived the Stoke's Law which gives the fluid resistance, or drags force, experienced by spherical objects when moving slowly in a viscous fluid:

$$f = kv = (6\pi\eta r)v$$

In this scenario, the magnitude off is proportional to the velocity **v** of the sphere, where k is a constant depends on the properties of the sphere and fluid: η is the fluid's viscosity, and r is the radius of the spherical object. Considering Newton's Second Law:

$$mg - kv - B = m\cdot\frac{dv}{dt}$$

When the forces acting on the body are balanced, acceleration is zero and the velocity reaches "terminal velocity" v_T:

$$v_T = \frac{(mg - B)}{k} = \frac{(mg - B)}{6\pi\eta r}$$

Therefore, after measuring the terminal velocity v_T, I can determine the value of liquid viscosity η.

3. Experimental Design

The purpose of this investigation is to determine the viscosity of certain liquid using Stoke's Law for the motion of a sphere in that liquid.

Apparatus:

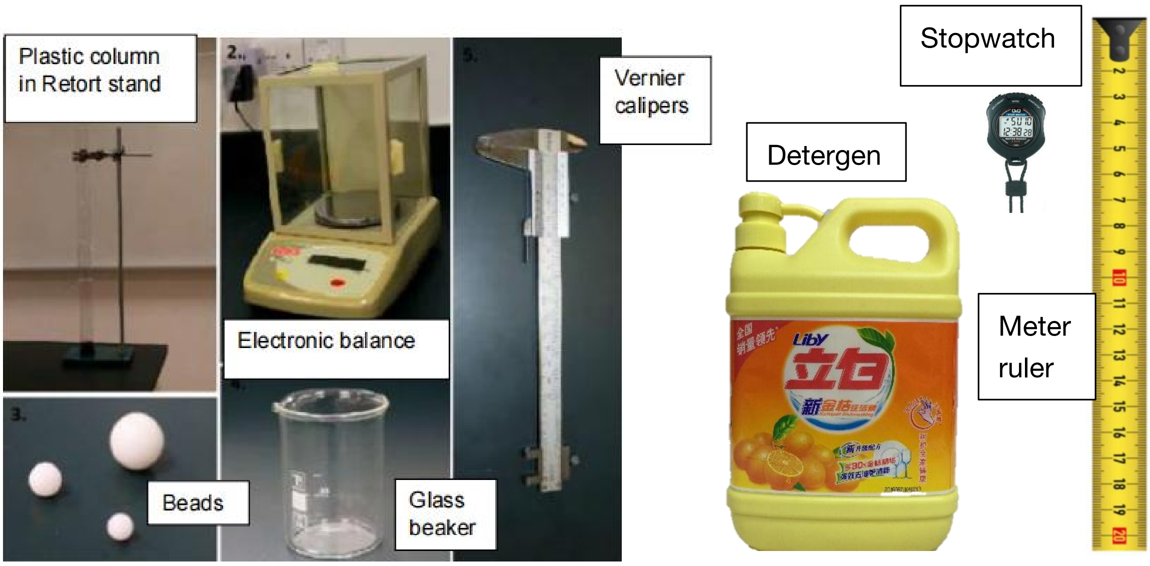


Figure 2: Apparatus used in this experiment

Procedure:

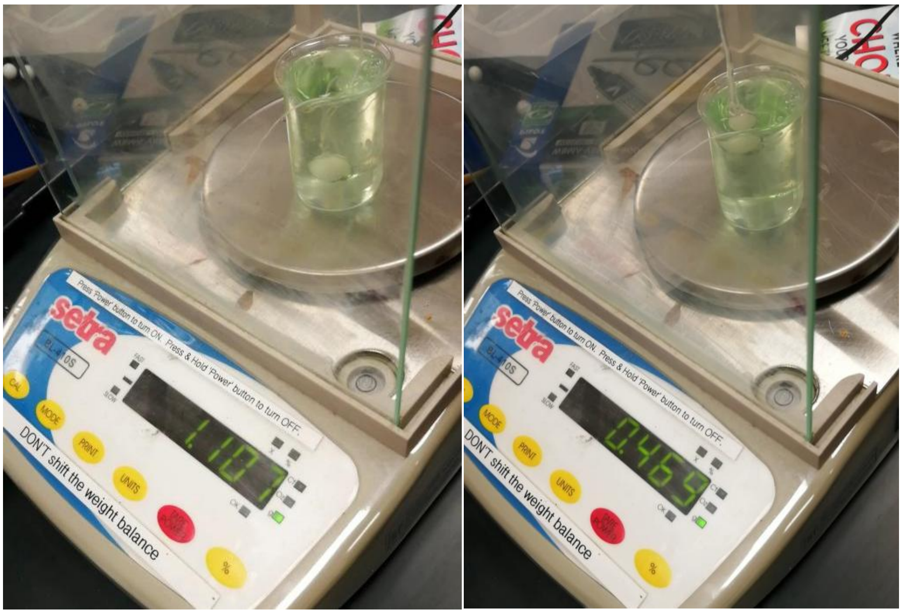
Part 1: Preliminary investigation:

This part is to verify the existence of buoyancy force and fluid resistance, and to determine the direction of these forces.

First, pour the detergent into the glass beaker and place it on the balance. Then press the Tare button. Suspend one bead from a string and immerse it in the detergent, such that the bead won't touch the bottom of the beaker. Record the reading on the balance, R1. Lower the bead to the bottom so that it sits on the bottom. Record the reading on the balance, R2. Lift the bead again, such that it is suspended in the detergent, record the reading on the balance R3.

Next, quickly drop then raise the bead in the detergent, observe the reading on the balance, R4.

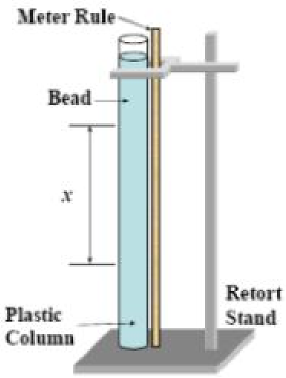
Lastly, release the bead to let it drop through the liquid. Observe the reading on the balance, R5.



Picture 2: Preliminary investigation

<u>Part 2: Measurement of the terminal velocity and the determination of the viscosity of the liquid</u>
First, weigh the beads. Then drop one bead into the center of the column, and measure the positions x of the bead at different times ***t***. After obtaining those data, I can determine the terminal velocity v_T of the beads. Repeat the previous step using beads of the three different sizes.

With these data, and taking detergent's density as 1.207 g / cm at room temperature,
I can determine the viscosity η of the detergent. After that, I will ccompare my value with the reference viscosity value of 1410 centipoise (cps). [1 centipoise = 1 millipascal-second (mPa s)]



Picture 3: Measurement of the terminal velocity and the determination of the viscosity of the liquid

<u>Experimental Variables</u>
In this part, **f** represents fluid resistance, **m** represents the mass of the beads, **B** represents buoyancy force, **r** represents the radius of beads, **R** represents the readings, **g** represents gravitational acceleration.

4. Data Collection

Part 1:

m(g)	R1(g)	R2(g)	R3(g)	R4(g)	R5(g)
0.942	0.747	0.964	0.724	Value decreasing	Value increasing

Table 1: Data collected in Part 1

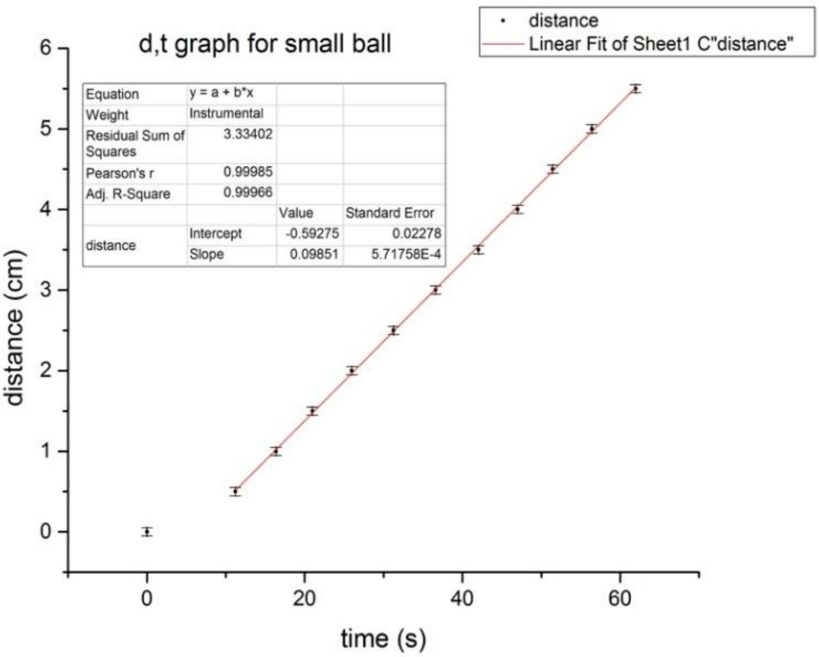
Because the electronic balance's unit is **g** (gram), the force including **f** (fluid resistance), **G** (gravity), and **B** (buoyancy) that act on the beads are expressed by the force values we measured multiplied by **g** (gravitational acceleration).

Part 2:

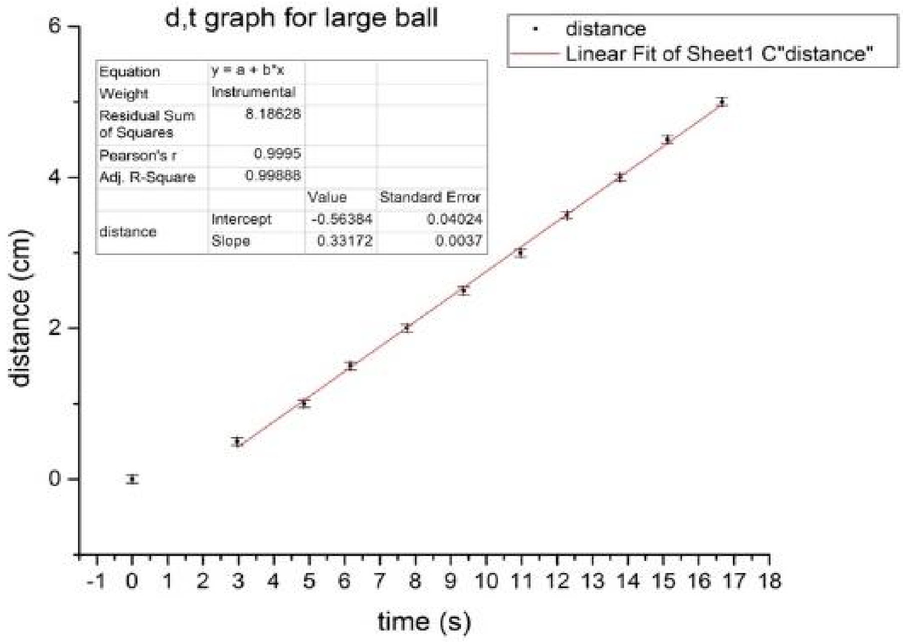
type	**mass**	**radius**	**type**	**mass**	**radius**	**type**	**mass**	**radius**
small	0.070g	2.29 mm	**medium**	0.179g	3.13 mm	**large**	0.939g	5.425 mm
Displacements (cm)		**Time (s)**	**Displacements (cm)**		**Time (s)**	**Displacements (cm)**		**Time (s)**
0		0	0		0	0		0
0.5		11.2	0.5		6.36	0.5		2.96
1		16.35	1		9.9	1		4.85
1.5		20.98	1.5		13.45	1.5		6.16
2		25.97	2		16.36	2		7.76
2.5		31.27	2.5		19.28	2.5		9.36
3		36.59	3		22.07	3		10.97
3.5		42.03	3.5		25.21	3.5		12.28
4		47	4		28.21	4		13.78
4.5		51.44	4.5		31.24	4.5		15.12
5		56.4	5		34.27	5		16.66
5.5		61.95						

Table 2: Data collected in Part 2

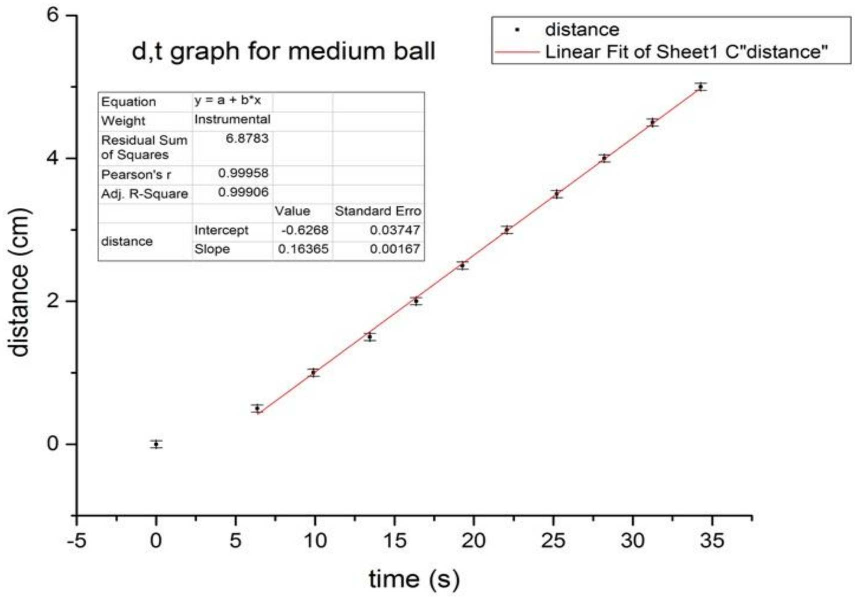
5. Photos and Graphs



Graph 1: Distance-Time graph for small ball



Graph 2: Distance-Time graph for large ball



Graph 3: Distance-Time graph for medium ball

6. Analysis

As for part 1, I place the beaker with detergent on the electronic balance. After the reading stabilizes, I press the Tare button so that the weight of the glass beaker and detergent can be ignored. Theoretically, R1 equals to B/g, R2 equals to m, R3 equals to B/g (same as R1), R4 equals to (B – f)/g, and R5 equals to (B + f)/g.

For the first step, I hold the string to immerse and suspend the bead in detergent. And I don't hold the string too tight because it will lead to significant error.

For the second step, I simply let the bead sit on the bottom. I find that R2 is slightly larger than **m** we measured before.

For the third step, the R3 is supposed to be equaled to R1. However, R3 that I observe is slightly smaller than R1.

For the fourth step, I can observe that when the bead quickly sinks, the reading increases, and when it is being raised, the reading rapidly decreases. As soon as I stop raising it, the reading goes back.

In the fifth step, R5 gradually increases and when it reaches a certain value, it stops.

In Part 2 of the experiment, I want to calculate the viscosity (η)of the detergent. First of all, I set 10~11 checkpoints along the column according to a certain displacement. Then, I release the three different beads separately above the surface of the liquid and record the time passing each check point. After that, I plot the graph with the software *Origin.* Based on the graphs, I am able to obtain terminal velocities, which show a constant slope. Finally, I calculate the viscosity (η) of the liquid by using this equation:

$$v = \frac{(mg - B)}{6\pi\eta r}$$

$$\eta = \frac{(mg - B)}{6\pi r v}$$

$$\eta = \frac{(mg - \rho g \cdot \frac{4\pi r^3}{3})}{6\pi r v}$$

After calculation, the terminal velocity of the small bead is **0.09851**, for medium bead, it's **0.16365**, for large bead it's **0.33172** (unit: cm/s).

In this experiment, I use the international system of units (SI), then the results of viscosity are:
Small bead: 2139.684 (mPa*s)
Medium bead: 2432.471 (mPa*s)
Large bead: 3807.005 (mPa*s)

7. Error Analysis & Source of Error

In part 1, the first and third steps, R1 and R3 should be equaled to B/g. However, as the string gives an upward tension(**T**) to the detergent near the string, R1 and R3 are actually equal to (B-T)/g, which is smaller than the theoretical value.

Comparing **m** and R2, they should be the same, but I can observe that R2 **>m**. That is because the mass of the string is neglected in this experiment, but R2 equals to **m**(bead)+mass of string, which is larger than **m**.

R1 and R3 are supposed to be the same, but it's observed that R3 <R1. That is because when I lift the bead, the rope lifts some detergent that attaches to the string out of the liquid surface. When there is less detergent in the beaker, R3 <R1.

Because I use tape to connect the string and beads, beads' shapes are changed, thus make the liquid resistance force larger.

In part 2, the theoretic value of η is given as 1410 (mPa*s).

The relative error is expressed as:

$$relative\ error = \frac{|factual\ value - theoretic\ value|}{theoretic\ value}$$

Through calculation, I get to know that the relative error for the small bead is 51.75%, for medium bead it's 72.53%, for large bead, it's 170.00%.

As for the measurement of time. I record videos that contain the whole process of dropping beads, and I determine the timings by watching these videos. However, it is difficult to precisely determine the exact time that beads reach the checkpoints. Also, my eyesight might not be perfectly vertical to the meter rule's scale lines, which causes squint. Additionally, it is not easy to determine when the beads reach terminal velocities as the time interval gradually becomes constant.

More importantly, Stoke's Law isn't very suitable for this experiment scenario. Stoke's Law's ideal condition is "unbounded incompressible Newtonian fluid". However, the plastic column is bounded with a relatively small diameter, and sometimes I drop the bead to close to the wall of the column (not perfectly in the centre), which can cause a greater drag to the beads.

All these above lead to the very large value of η, compared with the theoretic one.

8. Uncertainty

In part 1, the electronic balance's precision value is 0.001g. Therefore, the uncertainty is 0.0005g.

In part 2, the uncertainty of time is 0.005s because stop watch's precision value is 0.005s. The uncertainty of displacement is 0.05cm because the precision value of the meter ruler is 0.1cm. After that, the uncertainty of beads' radius is 0.005mm because the precision value of the Vernier calipers is 0.01mm. The uncertainty values are very small compared to our data, so it only contributes to a relatively small portion of the error.

9. Further Improvement

In part 1, the value of R1 and R3 should be measured as soon as the beads are suspended before the string tightens. And lighter string could be used to lower the error in R2 measurement. Meanwhile, I should use some tools to clear the detergent on the string back to the beaker in the measurement of R3. Beads and string should be joined by tape. Instead, I should use super glue so that the shapes of the beads can remain almost the same.

In part 2, I can determine whether the beads reach the checkpoints by observing the bottom of the beads. Also, I may adopt columns with much larger diameters and try to drop the beads right in the center of the column. Additionally, I can use sensors connected to a computer to track the beads' motion, which will be more accurate.

10. Conclusion

The ideal situation of applying Stoke's law is in unbounded incompressible Newtonian fluid". Otherwise, there will be significant errors. In very thick liquids, the string will also give the liquid tension. When the bead drops freely, its velocity increases to a certain value when the external force equals to zero because the resisting force is directly proportional to the velocity.